PHYSICS NYB-10/11 Winter 2007

Lecture 11: Direct current circuits

Instructor: Jérémie Vinet

Marianopolis College.

Review

- $I = \frac{dQ}{dt} = nqAv_d$
- $J = \frac{I}{A} = nqv_d$
- $\vec{J} = \sigma\vec{E} = \frac{1}{\rho}\vec{E}$
- $R = \rho\frac{l}{A}$
- $I = \frac{\Delta V}{R}$
- $P = \Delta V I = RI^2 = \frac{\Delta V^2}{R}$
- The resistance in an electrical device is constant, its power output is not

Review

Knowing that Hydro-Quebec charges $0,0522\$$ per kilowatt-hour of energy you use, how much money do you save in a year by using an energy efficient light-bulb which gives off the same amount of light as a standard 100 W lightbulb by drawing only 23 W of power?

Review

First, you need to know that a kilowatt-hour is the amount of energy drawn in one hour if we are drawing it at a rate of 1 kW, i.e. 1000 J/s. Therefore, 1 kWh $= 3.6 \times 10^6$ J.

Now a lightbulb in your house won't be turned on all the time, but let's assume it is on for an average of 5 hours a day. This means that a 100 W lightbulb will draw $100 \times 5 \times 3600 \times 365 = 6.57 \times 10^8$ J of energy during one year. This is 182.5 kWh, which will cost you $9,53\$$. The 23 W bulb will cost you 23% of this, namely $2,19\$$, saving you $7,34\$$.

Review

This might not seem like much, but consider the fact that you have many lightbulbs in a house. (I counted 16 in my house, but might have missed a few). Also, the rate used here is the one for the first 30 kWh per day. Anything exceeding that is charged at $0,0683\$$ per kWh, and usually anything you save will save you that rate. So for a household, the savings can easily reach between 100 and 200 dollars a year.

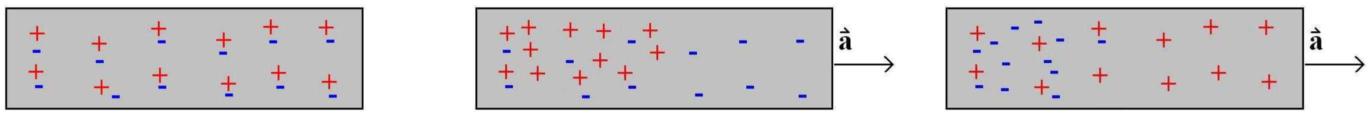
Experimental evidence

We've been talking about how electrons are free to move inside metals for a while now. You might be wondering *how we know this*...

In physics, all our knowledge comes from <u>experiment and observation</u>!!!

In 1916, Tolman and Stewart accelerated metal rods to figure out whether positive or negative charges are free to move. If we give a large acceleration to a rod, the charges that are free to move will be "thrown" towards the back of the rod. We can then check whether the back end of the rod is positive or negative.

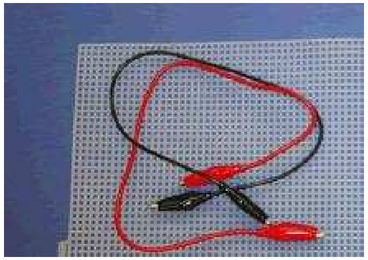
Tolman-Stewart experiment



- A metal rod has free charges. We give it a large acceleration
- If the free charges are positive, the back end of the rod should be positive
- If the free charges are negative, the back end of the rod should be negative
- The results proved that electrons are the charge carriers in a metal.

Resistance in wires

Knowing the conductivity of copper is $5.88 \times 10^7\ \Omega^{-1}\cdot\text{m}^{-1}$, estimate the resistance of a typical copper wire you use in the lab. Compare this to the typical value for resistors you use in the lab. Is it a good approximation to say that the resistance in the wires of a circuit is zero?

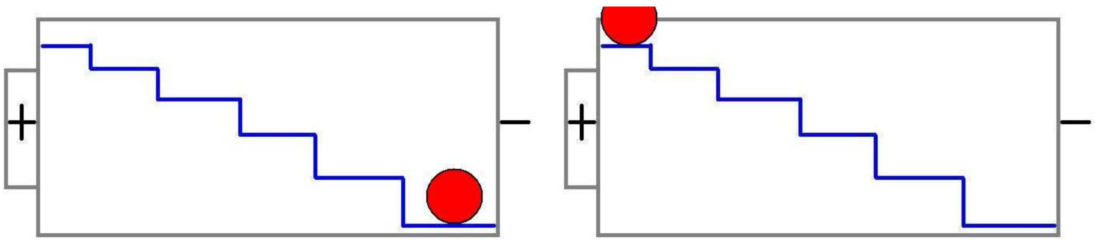


Let's take the length of the wire to be around 20 cm. The radius is on the order of around 0.5 mm. We can then use $R = \rho\frac{l}{A} = \frac{1}{\sigma}\frac{l}{\pi r^2}$ to find that the resistance of the wire is $R = 4.33$ mΩ. This is much smaller than the typical resitors we use in the lab, which are on the order of 1 Ω.

Batteries

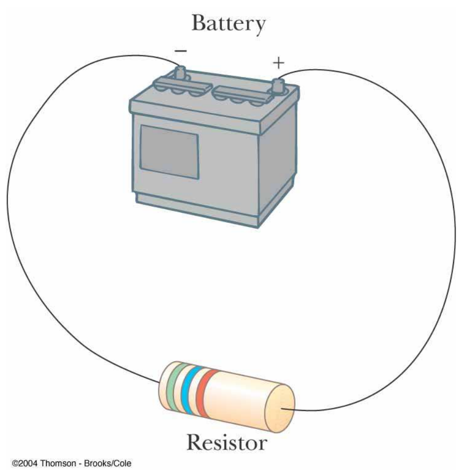
Recall from last lecture...

- A battery is a source of constant potential difference
- It takes charges and does work on them, giving them potential energy
- This energy comes from chemical energy in the battery, which is limited
- Therefore batteries eventually run out of energy

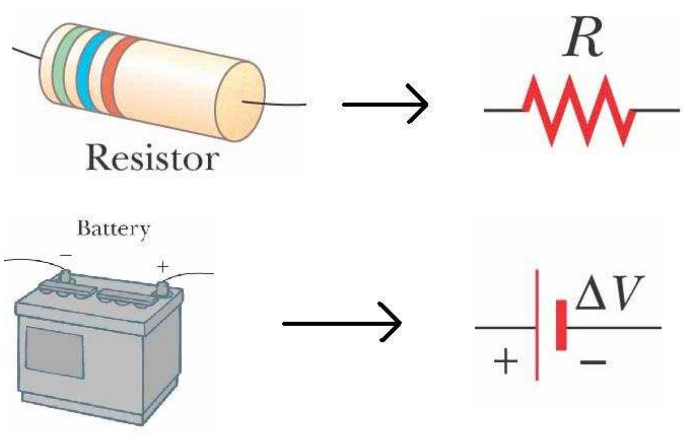


The simplest circuit

The simplest circuit we can make is to just connect a resistor to the terminals of a battery.



Before going further, we'll define more convenient ways of representing components in a circuit:



Conservation of energy in a circuit

What is the potential energy of an electron at the negative end of a 9 V battery? What is its potential energy at the positive end?

The potential energy is $U_e = qV = -eV$. We'll take the positive end to be at zero potential, so the negative end is at $V = -9$ V. This means the potential energy of the electron is zero at the positive end, and $-1.6 \times 10^{-19} \times (-9) = 1.44 \times 10^{-18}$ J at the negative end.

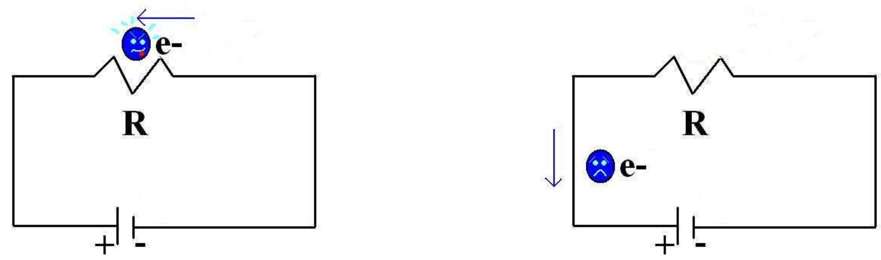
The electron's potential energy decreases. Energy is always conserved. Where does this energy go as the electron goes from the negative end to the positive end???

The potential energy gets converted into some other type of energy as the electron goes through a resistor.

Conservation of energy in a circuit

- The charges on one side of a battery (or power socket, or generator) have a lot of potential energy.
- The travel down the wire, losing very little energy, since the wire has very little resistance.
- Then they encounter the resistor, where the resistance is large.

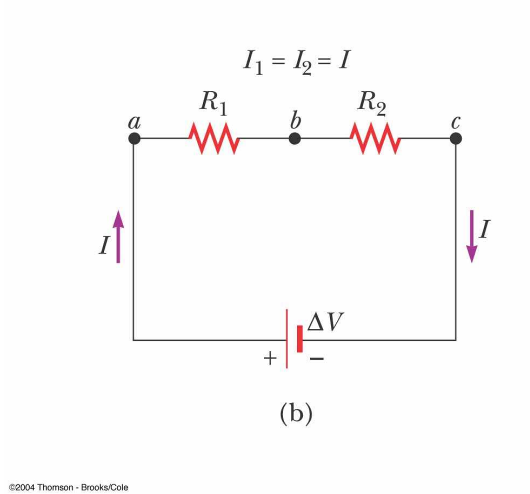
Conservation of energy in a circuit



- They lose their potential energy, which is converted into something else.
  - light, heat, mechanical energy, etc...
- The electrons reach the other side of the battery (or power socket, or generator) with no potential energy.
- They are then given back some energy by the battery (or power socket, or generator) and repeat the process.

Multiple resistor circuits

Resistors in series:



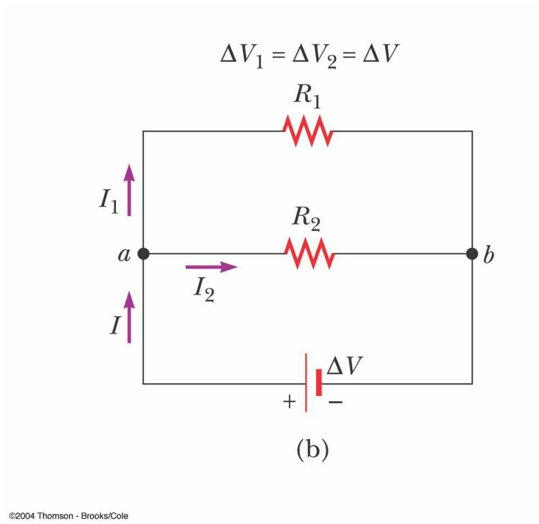
The key point concerning resistors in series is that the current through them is equal, while the total potential difference is the sum of the potential differences at each resistor.

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2 = R_1 I_1 + R_2 I_2 = (R_1 + R_2) I = R_{eq} I$$

$$\boxed{\Rightarrow R_{eq} = R_1 + R_2 + \cdots}$$

Multiple resistor circuits

Resistors in parallel:



The key point concerning resistors in parallel is that the potential difference through both is the same, while the total current is the sum of the currents through each resistor.

$$\Delta V = R_{eq} I = R_{eq}(I_1 + I_2) = R_{eq}\left(\frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}\right) = \Delta V R_{eq}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\boxed{\Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots}$$

Series vs. parallel

When resistors are in series, the equivalent resistance is clearly greater than the resistance of each resistor, since

$$R_{eq} = \sum R_i \quad .$$

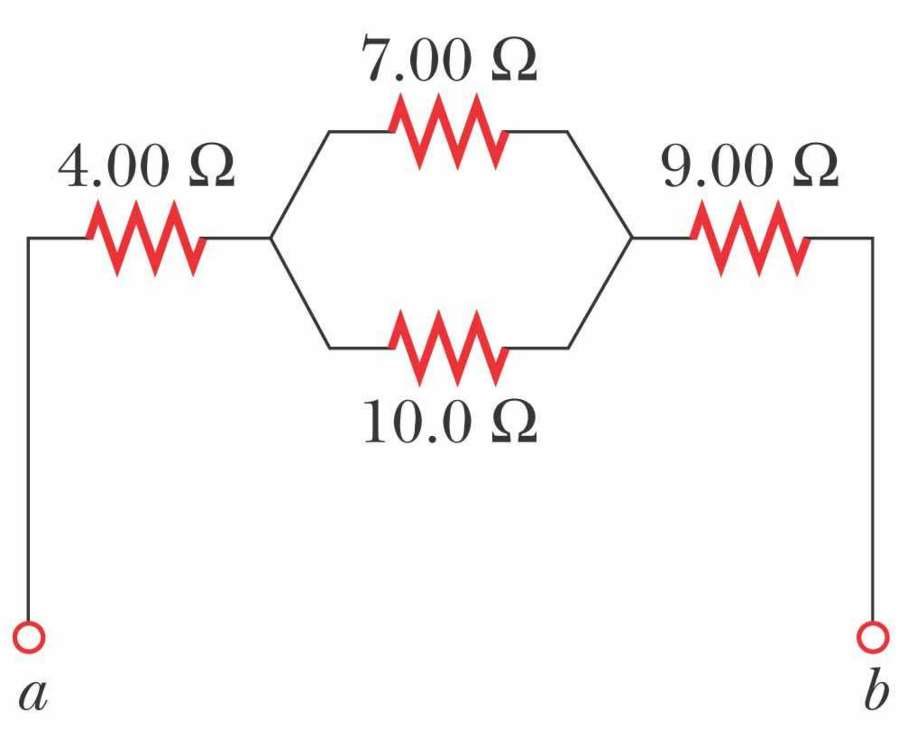
When resistors are in parallel, is the equivalent resistance greater or less than the resistance of each resistor? Or does it depend on the actual values?

$$\begin{aligned} R_{eq} &= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \\ &= \frac{R_1}{1 + \frac{R_1}{R_2}} < R_1; \qquad = \frac{R_2}{\frac{R_2}{R_1} + 1} < R_2 \end{aligned}$$

so the equivalent resistance is always *less* than either resistance in parallel.

Example

What's the effective resistance in this circuit? If $\Delta V_{ab} = 34$ V, what is the current in each resistor? Which resistor is drawing the most power? The least?



Example

First off, notice that when you have two resistors in parallel, you can rewrite $R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$ as $R_{eq} = \frac{R_1 R_2}{R_1+R_2}$, which makes calculating it simpler.

The two resistors in parallel have $R_{par} = \frac{10\times 7}{10+7} = \frac{70}{17}\ \Omega$. This equivalent resistance is in series with the other two so that $R_{eq} = 4 + \frac{70}{17} + 9 = 17.1\ \Omega$.

Now we can use $I = \frac{\Delta V}{R}$ to find $I = 1.99$ A, which is the current in the 4 and 9 Ω resistors. It is also the total current in the parallel resistors, so that the potential drop across this resistor is $\Delta V_{par} = IR_{par} = 8.2$ V. This is the potential drop at each of the two resistors, so that $I_7 = \frac{8.2}{7} = 1.17$ A, and $I_{10} = \frac{8.2}{10}$ = 0.82 A, for a total of 1.99 A, as expected.

Example

The power in each resistor is $P = \Delta V I = RI^2$. Using the values we found above, we find that $P_4 = 4 \times (1.99)^2 = 15.84$ W, $P_9 = 9 \times (1.99)^2 = 35.64$ W, $P_{10} = 10 \times (0.82)^2 = 6.72$ W and $P_7 = 7 \times (1.17)^2 = 9.58$ W. So the 9 Ω resistor is drawing the most power, while the 10 Ω one the least. Note also that the total power is $15.84 + 35.64 + 6.72 + 9.58 = 67.78$ W, while if we had used the total current with the equivalent resistance, we would get $P = R_{eq} I_{tot}^2 = 17.1 \times (1.99)^2 = 67.7$ W, as it should be. (The slight difference is due to rounding off multiple values).

Example

A tea kettle has a multiposition switch and two heating coils. When only one of the coils is switched on, the kettle brings a pot of water to a boil in a time Δt. When only the other coil is switched on, it takes a time $2\Delta t$ to bring the same amount of water to a boil. How much time would it take to boil the same amount of water if the two coils are switched on in a series combination, and in a parallel combination?

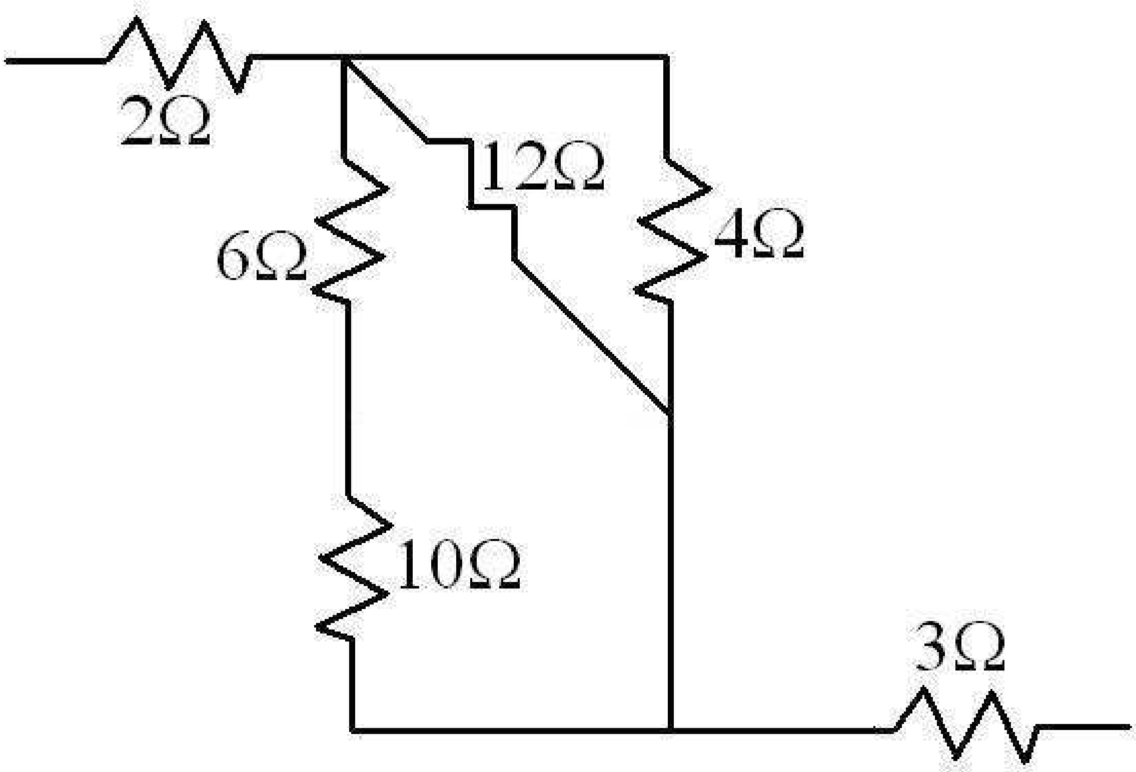
Example

A tea kettle will be plugged into a standard electrical outlet, so the potential difference in all cases here is $\Delta V = 120$ V. The amount of time it will take to boil the pot is inversely proportional to the power delivered to the coil, $P = \frac{\Delta V^2}{R} = \frac{\Delta E}{\Delta t}$. From this, since the time to boil the pot with coil 2 is twice as big as the time to boil the pot with coil 1, we find that $2R_1 = R_2$.

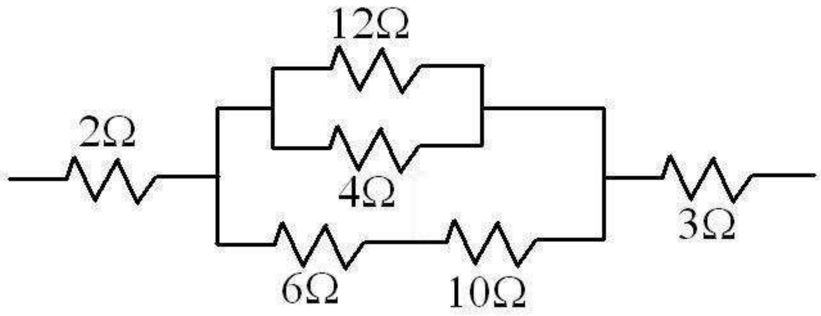
If the coils are in series, we'll have $P = \frac{\Delta V^2}{R_1+R_2} = \frac{\Delta V^2}{3R_1}$, so the time will be $3\Delta t$. If they are in parallel, then $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{3}{2R_1}$ so the time will be $\frac{2}{3}\Delta t$.

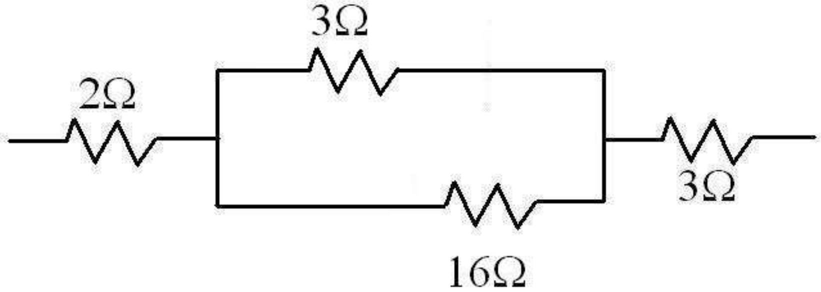
Example

Solve this circuit completely, where the potential difference between the ends in 143 V.

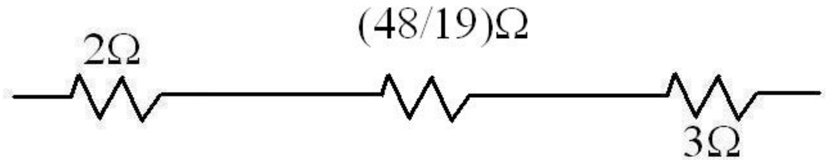


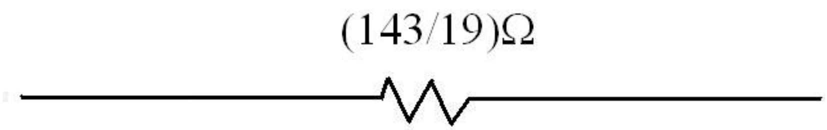
Example



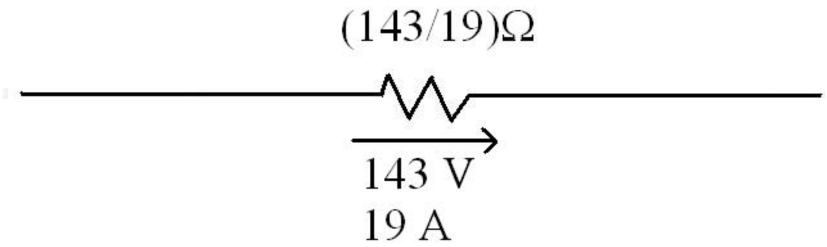


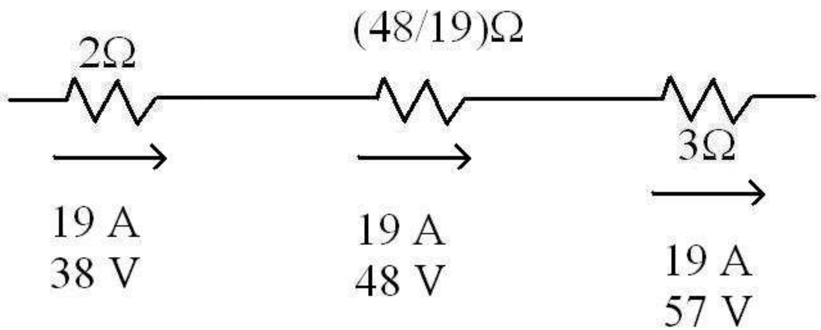
Example



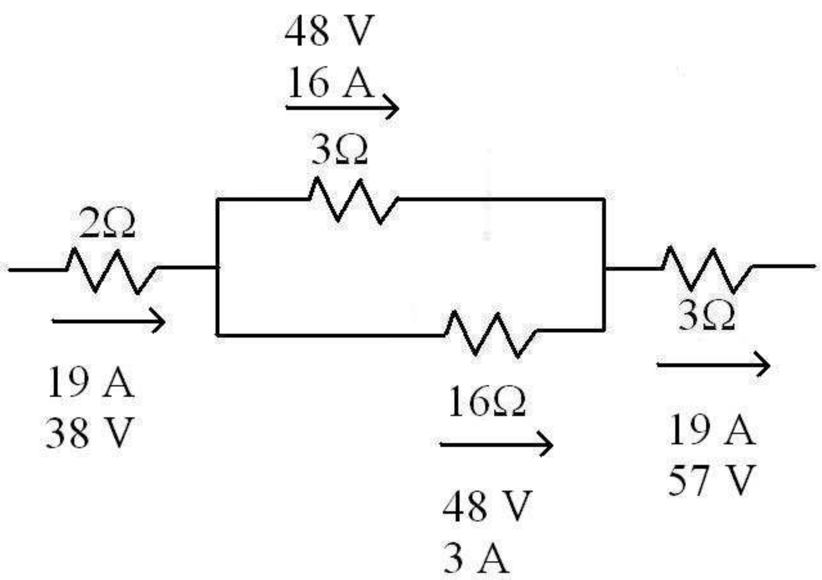


Example

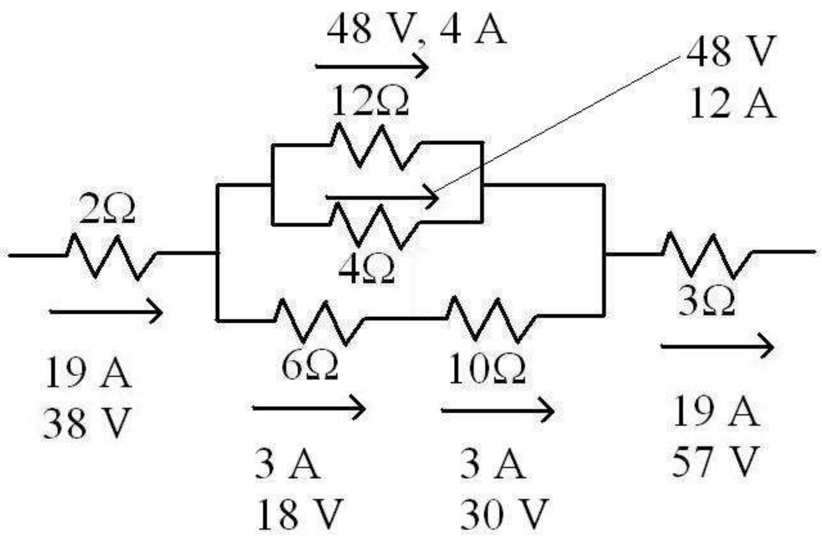




Example



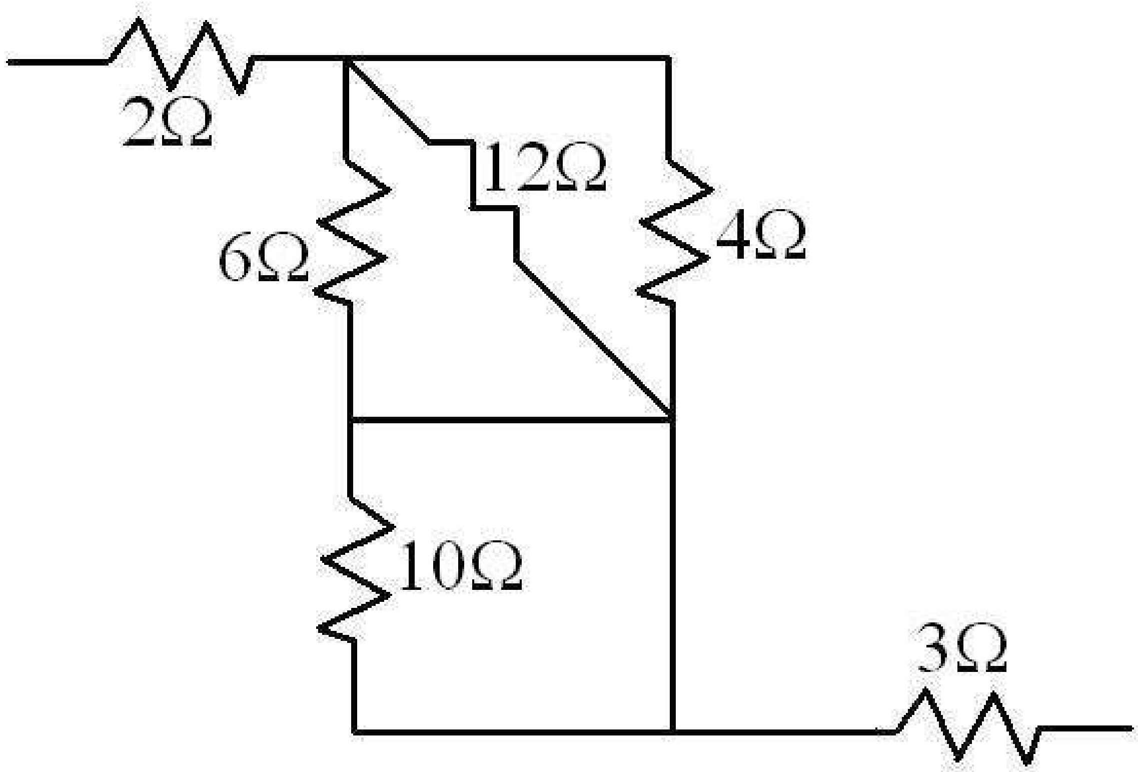
48 V
16 A
3Ω
2Ω
3Ω
19 A
38 V
16Ω
19 A
57 V
48 V
3 A



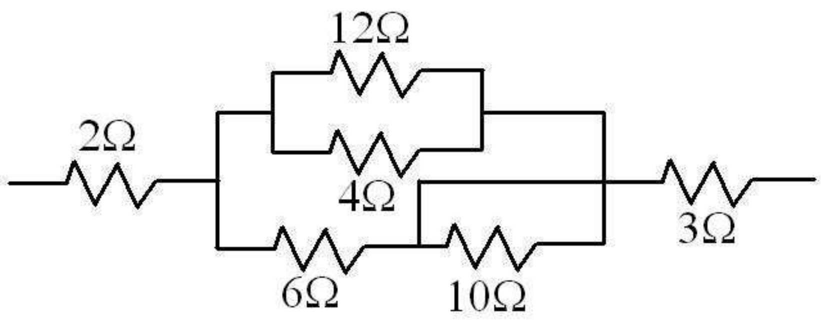
48 V, 4 A
48 V
12 A
12Ω
2Ω
4Ω
3Ω
19 A
38 V
6Ω
10Ω
19 A
57 V
3 A
18 V
3 A
30 V

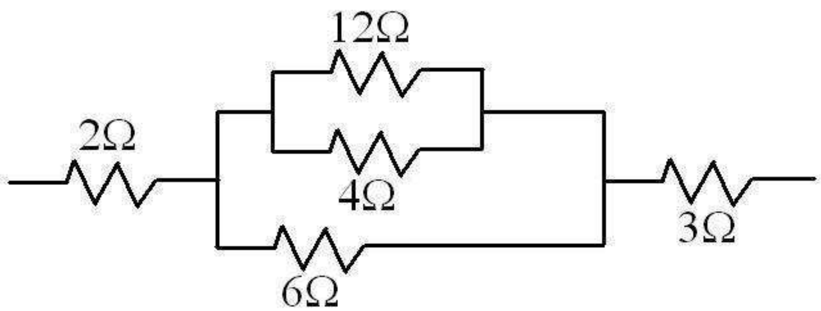
Example

Now solve this circuit completely, where the potential difference between the ends is 140 V.

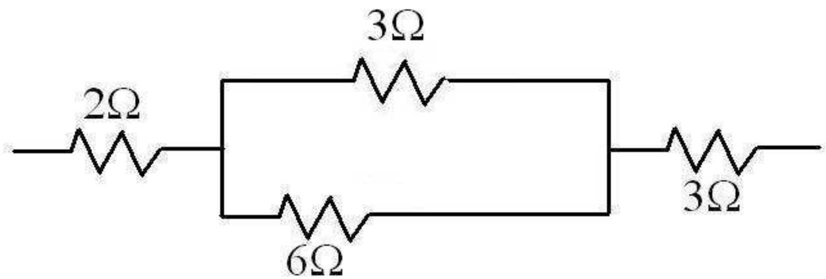


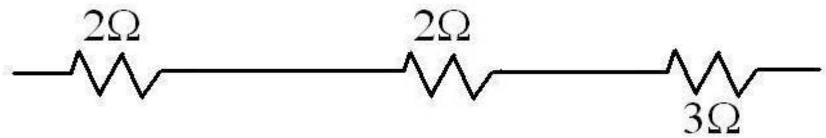
Example



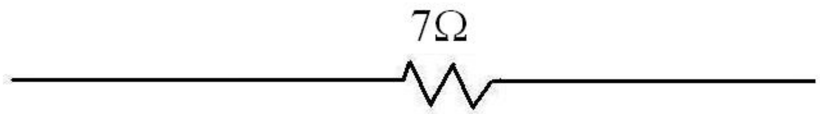


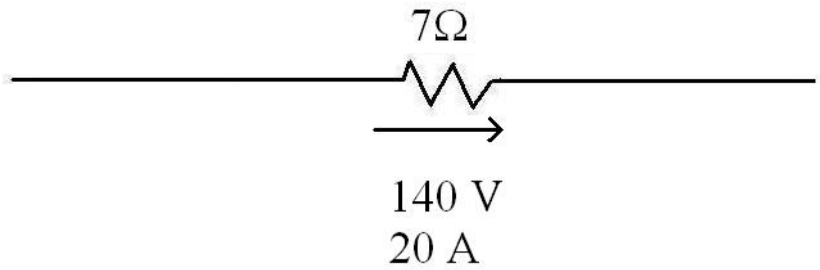
Example



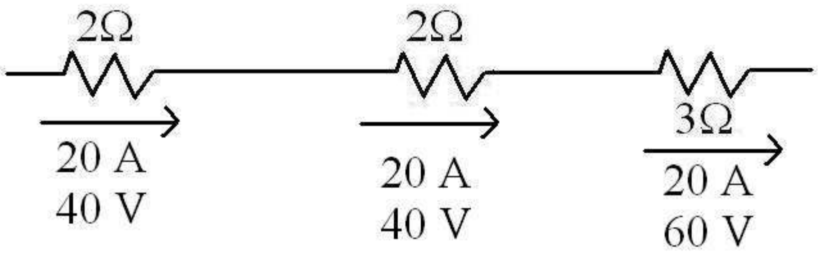


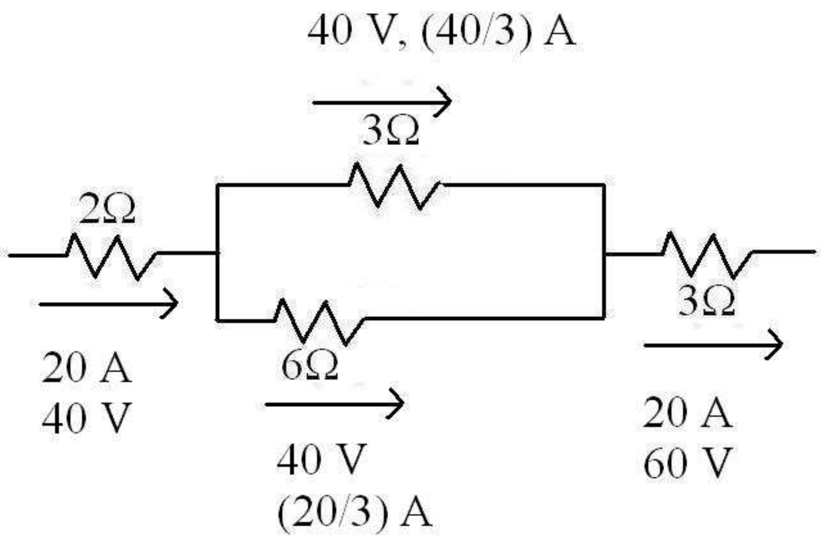
Example



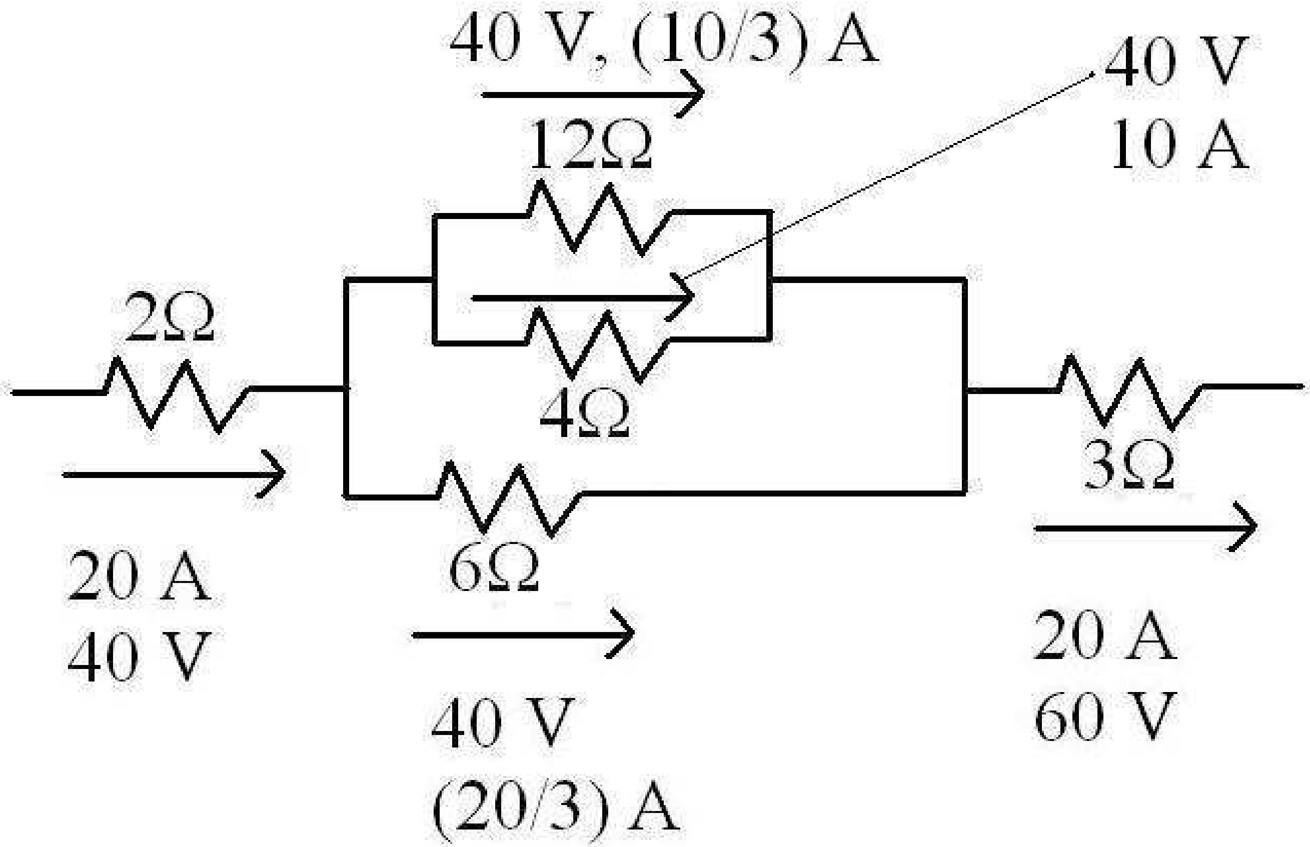


Example


2Ω
2Ω
3Ω
20 A
40 V
20 A
40 V
20 A
60 V


40 V, (40/3) A
3Ω
2Ω
3Ω
6Ω
20 A
40 V
40 V
(20/3) A
20 A
60 V

Example

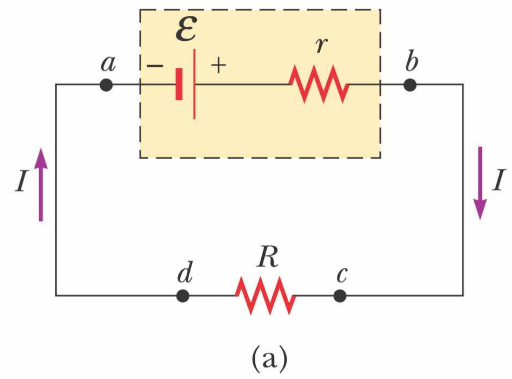


Electromotive force

The potential difference through which the battery lifts the charges is called *electromotive force*, emf, or $\mathcal{E}$. This is not a good term, because the emf is <u>NOT A FORCE</u>!!! But its the accepted term, so we'll have to use it anyway...

Now from what we've learned about Ohm's law, does this mean we can find the current through the resistor from $\mathcal{E} = RI$?

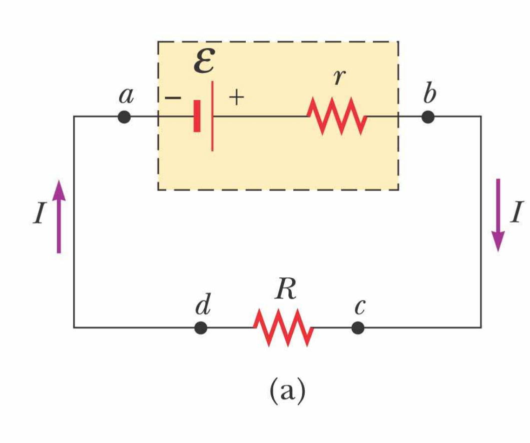
Electromotive force



No! Actually, any battery has an *internal resistance* r which leads to a potential drop Ir for charges before they even leave the battery. So the actual potential difference between the terminals is $\Delta V = \mathcal{E} - Ir$. So

$$\begin{aligned} \mathcal{E} &= IR + Ir \\ I &= \frac{\mathcal{E}}{R+r} = \frac{\Delta V}{R} \end{aligned}$$

Power delivered by a battery



We know that $P = \dfrac{\Delta V}{I} = I^2 R$. So for a battery with emf $\mathcal{E}$, the total power delivered by the battery is

$$I\mathcal{E} = I^2 R + I^2 r$$

so that only a certain proportion of the battery's power is delivered to the external resistor R. This fraction is

$$\frac{P_R}{P_{tot}} = \frac{I^2 R}{I^2 R + I^2 r} = \frac{1}{1 + \frac{r}{R}}$$

Examples

You have a flashlight with two 1.50 V batteries with their positive terminals in the same direction. One of them has internal resistance $r = 0.255\ \Omega$ and the other has $r = 0.153\ \Omega$. When you turn it on, there is a current $I = 600$ mA in the lamp. What is the lamp's resistance? What fraction of the battery's power output is going to the lamp? As time goes by, the batteries' internal resistances increase. What happens then?

Examples

The total emf is twice 1.5. We know that $\mathcal{E} = IR_{tot} = I(R + r_1 + r_2)$. This means that $\frac{\mathcal{E}}{I} - r_1 - r_2 = R$, so that $R = 4.59\ \Omega$.

The fraction of the power that actually goes to the lamp is $\frac{P_{lamp}}{P_{tot}} = \frac{RI^2}{(R+r_1+r_2)I^2} = \frac{R}{R+r_1+r_2} = 91.8\%$.

As time goes by, the internal resistance increases, and the total power $P_{tot} = \frac{\mathcal{E}^2}{R_{tot}}$ decreases, while the fraction $\frac{P_{lamp}}{P_{tot}} = \frac{R}{R+r_1+r_2}$ also decreases, so the lamp will get dimmer and dimmer.

Examples

For a battery of fixed emf $\mathcal{E}$ and internal resistance r, for what value of R is the power delivered to the external resistance maximum?

We know that $\mathcal{E} = (R + r)I$, so $I = \frac{\mathcal{E}}{R+r}$. The power through the external resistor is $P = RI^2 = \frac{R\mathcal{E}^2}{(R+r)^2}$. We want to find the value of R for which P is maximized. This happens when the derivative of P with respect to R vanishes. So we want to find when $\frac{d\left(\frac{R\mathcal{E}^2}{(R+r)^2}\right)}{dR} = 0$. This happens when $\frac{\mathcal{E}^2(r-R)}{(R+r)^3} = 0$, i.e. when $R = r$.

Reducing internal resistance

Suppose you want to have a 1.5 V potential difference in a given circuit. You have a bunch of batteries with emf $\mathcal{E} = 1.5$ V, but you know that since they have internal resistance, the actual terminal potential difference will be lower than that. What can you do to reduce the effective internal resistance?

You can connect a number of these batteries in parallel. The effective emf is still 1.5 V, but the effective internal resistance will now be

$$r_{eff} = \frac{1}{\sum_i^n \frac{1}{r_i}} = \frac{1}{\frac{n}{r_i}} = \frac{r_i}{n}$$

What to read for next lecture

- 28.3